

## Myths and rules of thumb in retirement income modelling appendix

### **Introduction**

'*Myths and rules of thumb in retirement income*' is the second stage of a research project sponsored by State Street Global Advisors. It builds on the findings from the first qualitative research stage that explored how individuals approaching retirement might use their Defined Contribution (DC) pension pots. This appendix to the report provides details of the modelling approach, the full report can be downloaded from the PPI website [www.pensionspolicyinstitute.org.uk](http://www.pensionspolicyinstitute.org.uk).

The report is part of the PPI's Transitions to Retirement research, a series of major reports exploring how people access pension savings in light of the new pension freedoms. The research series is sponsored by Age UK, The Investment Association, Partnership, The Pensions Advisory Service (TPAS), The Pensions Regulator (TPR), The People's Pension and Fidelity.

### **Stochastic Individual Modelling**

The modelling was created using the PPI's individual model. The PPI's individual model uses individual characteristics and working patterns to project income in retirement from private pensions, state pensions and other benefits for hypothetical individuals.<sup>1</sup>

The model was run assuming 1,000 different economic scenarios from the PPI's economic scenario generator. For each scenario, values are generated for CPI, earnings, gilt return and equity return. The actual value for each variable will vary around the median (which are defined in table A1). Using these variables, outcomes are generated within the PPI's individual model.

### **Economic scenarios**

This section provides a description of the model used to generate the economic scenarios for this project.

The model is based upon a combination of PPI economic assumptions and analysis of historical data. Table A1 summarises: the risk factors that were modelled; the sources of historical data used and; the PPI's long-term economic assumptions.

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<sup>1</sup> For more information on the Individual Model, see PPI (2003)

**Table A1: Model risk factors**

Abbreviation	Description
	<b>Source of historical data</b>
	<b>Long term assumptions</b>
G	Nominal GDP. ONS quarterly data from 30/06/1955 to present. <sup>2</sup> Annual GDP growth of 4.0%
P	CPI. ONS monthly data from 29/02/1988 to present. <sup>3</sup> Data from 31/01/1950 to 31/01/1989 derived from ONS RPI data using the methodology described by O’Neill and Ralph <sup>4</sup> . Annual CPI growth of 2.0%
Y <sup>l</sup>	Long term yields. End of month FTSE Actuaries 15 Year Gilts Index from 30/11/1998 to present. <sup>5</sup> Low coupon 15 year gilts yields from 31/12/1975 to 31/10/1998. <sup>6</sup> Nominal return on gilts of 4%
S	Stock returns. End of month FTSE All share total return index from 31/12/1985 to present. <sup>7</sup> Nominal return on equities of 7%

Using these variables, a six dimensional process,  $x_t$  is defined.

$$x_t = \begin{bmatrix} \ln G_t - \ln G_{t-12} \\ \ln(P_t - \ln P_{t-12} + 0.02) \\ \ln W_t - \ln W_{t-12} \\ \ln(e^{Y_t^l} - 1) \\ \ln(e^{Y_t^s} - 1) \\ \ln S_t \end{bmatrix}$$

Where t denotes time in months.

<sup>2</sup> Source Bloomberg L.P

<sup>3</sup> Source Bloomberg L.P

<sup>4</sup> Robert O’Neill and Jeff Ralph, Office for National Statistics (2013)

<sup>5</sup> Source Bloomberg L.P

<sup>6</sup> Data from the Heriot-Watt/Institute and Faculty of Actuaries Gilt Database

<sup>7</sup> Source Bloomberg L.P

The development of the vector  $x_t$  is modelled by the first order stochastic difference equation:

$$\Delta x_t = Ax_{t-1} + a + \varepsilon_t$$

Where  $A$  is a 6 by 6 matrix,  $a$  is a six dimensional vector and  $\varepsilon_t$  are independent multivariate Gaussian random variables with zero mean. The values of  $A$  and  $a$  and the volatilities and correlation of the  $\varepsilon_t$  are given in table A2. The matrix  $A$  and the covariance matrix of the  $\varepsilon_t$  were determined by calibrating against the historical data. The coefficients of  $a$  were then selected to match the long term economic assumptions.

It follows that the values of  $x_t$  will have a multivariate normal distribution. Simulated investment returns will, however, be non-Gaussian partly because of the nonlinear transformations above. Moreover, the yields are nonlinearly related to bond investments.

The first component and third components of  $x_t$  give the annual growth rates of GDP and wages, respectively. The fourth and fifth components are transformed yields. The transformation applied ensures that the yields are always positive in simulations. Similarly the second component gives a transformed growth rate of CPI. In this case, the transformation applied ensures that inflation never drops below -2% in the simulations. This figure was selected to be twice the maximum rate of deflation ever found in the historical data. More sophisticated transformations of the CPI that allow for arbitrarily negative deflation could be considered instead, but seem unnecessary for the purposes of this paper.

**Table A2: Model parameters**

	G	P	W	Y <sup>l</sup>	Y <sup>s</sup>	S	
The matrix $A$	G	0.0000	-0.0026	0.0000	0.0010	-0.0006	0.0000
	P	0.0000	-0.0383	0.3936	0.0000	0.0000	0.0000
	W	0.1028	0.0000	-0.3759	-0.0010	0.0020	0.0000
	Y <sup>l</sup>	0.0000	0.0000	0.0000	-0.0055	0.0000	0.0000
	Y <sup>s</sup>	6.4361	0.0000	0.0000	0.0000	-0.0348	0.0000
	S	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		G	P	W	Y <sup>l</sup>	Y <sup>s</sup>	S
The vector $a'$	-0.0101	-0.1406	0.0085	0.0220	-0.1190	0.0058	
	G	P	W	Y <sup>l</sup>	Y <sup>s</sup>	S	
Annual volatility of $\varepsilon_t$	0.41	0.09	1.20	1.34	1.25	0.73	
	G	P	W	Y <sup>l</sup>	Y <sup>s</sup>	S	
Correlation matrix of $\varepsilon_t$	G	1.00	-0.01	0.11	0.07	0.10	0.13
	P	-0.01	1.00	0.02	0.06	0.04	-0.04
	W	0.11	0.02	1.00	0.15	0.07	-0.02
	Y <sup>l</sup>	0.07	0.06	0.15	1.00	0.30	-0.12
	Y <sup>s</sup>	0.10	0.04	0.07	0.30	1.00	-0.12
	S	0.13	-0.04	-0.02	-0.12	-0.12	1.00

Monthly log-returns on bond and money market investments are given by

$$R_t^j = Y^j/12 - D^j \Delta Y_t^j \quad j = l, s$$

Where  $D$  is the duration of the investment class,  $D^l = 12.25$  and  $D^s = 0.125$ .

For a general reference on multivariate time series analysis see Lütkepohl<sup>8</sup>. Other applications of the modelling approach presented here can be found, for example, in Koivu, Pennanen and Ranne<sup>9</sup> and Aro and Pennanen (2005)<sup>10</sup>.

### **Limitations of analysis**

Care should be taken when interpreting the results in this report. In particular, one of the main limitations is that individuals are not considered to change their behaviour in response to investment performance. For example, if investments are performing poorly, an individual may choose to decrease their withdrawal rate and vice versa.

<sup>8</sup> Lütkepohl (2006)

<sup>9</sup>M.Koivu, T.Pennanen and A.Rann (2005)

<sup>10</sup> H.Aro and T.Pennanen (forthcoming)

Monte Carlo simulation can be a powerful tool when trying to gain an understanding of the distribution of possible future outcomes. However, in common with other projection techniques, it is highly dependent on the assumptions made about the future. In this case, the choice of distribution and parameters of the underlying variables, the investment returns of equities, gilts and cash are important to the results.

### **Modelling Assumptions**

The model was run on the following assumptions:

- The median return on equities is assumed to be 7% and the median return on gilts is assumed to be 4%.
- The pension pot is assumed to be invested 40% in gilts and 60% in equities.
- There is a 0.75% charge on drawdown.
- The pension pot is assumed to go through drawdown after taking a 25% tax-free lump sum.
- The individuals are assumed to retire at their state pension age of 65.
- CPI increase is assumed to be 2.0%.
- The average life expectancy is taken from the ONS cohort mortality for males and females aged 65.
- For drawdown, the rate is assumed to be the percentage of the initial pot uprated by CPI.